UG Program (under NEP 2020)
1st Semester Exam., 2025 (held in 2026)

MATHEMATICS

(Major)

Paper Code : MT101C

Full Marks : 60

Time : 3 hours

The figures in the margin indicate full marks for the questions

GROUP—A

1. Answer *any* **six** of the following questions : 2×6=12

(a) Is the function $f:\mathbb{R}\to\mathbb{R}$, defined by $f(x)=x^2$, a bijection? What modifications do you need to make in the domain and codomain to make it a bijection?

(b) State the contrapositive of the implication 'If A is finite, then A is countable'.

(c) Let $x, y > 0$. Show which is larger, $\sqrt{xy}$ or $\dfrac{2xy}{x+y}$.

(d) Let X be a nonempty set and $P(X)$ denotes the power set of X. If $(P(X), \subseteq)$ is a Poset, when will $\subseteq$ be a total order on $P(X)$?

(e) State Zorn's lemma.

(f) Find the equation whose roots are the squares of the roots of $x^3 - 4x + 2 = 0$.

(g) State *m*th power theorem.

(h) Construct a function $f : \mathbb{R} \to \mathbb{R}$ that is onto but not one-to-one.

GROUP—B

Answer **four** questions, taking one from each Unit

UNIT—I

2. (a) Negate the following statements :

(i) He is well-built as he is tall and muscular.

(ii) Ram likes a fruit if it is sweet or juicy.

(b) Give an example of an equivalence relation with proper justification.

(c) (i) Check whether $f : \mathbb{R} \to \mathbb{R}$ defined by $f(x) = \log x$ can be a bijection or not.

(ii) Also check $f : \mathbb{R} \to \mathbb{R}$ given by $f(x) = e^x$ is a bijection or not. What can you do to make it a bijection? (2+2)+4+(2+2)=12

3. (a) Show equivalence of the following :

$[d \to ((\sim a) \wedge b) \wedge c]$ and $\sim [(a \vee (\sim (b \wedge c))) \wedge d]$

(b) There are two shopping malls next to each other, one with sign board as 'Good items are not cheap'. And the second with sign board as 'Cheap items are not good'. Do they mean same? Justify.

(c) The relation R in a set A is defined by $2x + 3y = 30$, $x, y \in A$, where $A = \{0, 1, 2, 3, \ldots, 15\}$. Write down R and identify its type.

(d) Given that $f : R \to R$ defined by $f(x) = \tan x$. Show that f is not a mapping. 3+3+4+2=12

UNIT—II

4. (a) In a Poset $(P, \subseteq)$, define upper bound and least upper bound of a subset. Give suitable example.

(b) For the Poset $(P(\{1, 2, 3\}), \subseteq)$, find all maximal and minimal elements.

(c) Explain the equivalence between the well-ordering principle and mathematical induction.

(d) State Schroder-Bernstein theorem.

3+3+4+2=12

5. (a) Show that the set of all rational numbers Q is countable.

(b) What is a chain in a Poset? Give an example of a Poset in which every chain has an upper bound but the Poset has no greatest element.

(c) State both the Peano's axioms.

4+(1+3)+4=12

UNIT—III

6. (a) Prove that the imaginary roots of a polynomial equation with real coefficients occur in conjugate pairs.

(Continued)

(b) Find the equation whose roots are the squares of the differences of the roots of the cubic $x^3 - 7x - 6 = 0$.

(c) If a, b, c are three positive numbers, then show that

$$\left(\frac{a+b+c}{3}\right)^3 \geq a\left(\frac{b+c}{2}\right)^2$$

(d) For two positive real numbers a and b, show that $AM \geq GM$.

3+4+3+2=12

7. (a) If $a_1, a_2, a_3, \ldots, a_n$ are n positive numbers in AP, then show that

$$a_1^2 + a_2^2 + \ldots + a_n^2 > n\left(\frac{a_1 + a_n}{2}\right)^2$$

(b) If the equation

$$x^4 + ax^3 + bx^2 + cx + d = 0$$

has three equal roots, then show that each of them is equal to

$$\frac{6c - ab}{3a^2 - 8b}$$

(c) State the fundamental theorem of algebra.

(d) Find the remainder when $x^4 + 4x^2 - 9x + 21$ is divided by $2x - 7$.

4+4+2+2=12

(Turn Over)

UNIT—IV

8. (a) If α, β, γ are the roots of the equation $x^3 + px^2 + qx + r = 0$, then find the value of

$$\left(\frac{1}{\beta}+\frac{1}{\gamma}-\frac{1}{\alpha}\right)\left(\frac{1}{\gamma}+\frac{1}{\alpha}-\frac{1}{\beta}\right)\left(\frac{1}{\alpha}+\frac{1}{\beta}-\frac{1}{\gamma}\right)$$

(b) Solve the following equation by a suitable method :

$$x^4 + 5x^2 + 2x + 8 = 0$$

(c) Solve by Ferraris method

$$x^4 - 18x^2 + 32x - 15 = 0$$ 4+4+4=12

9. (a) Solve $x^3 - 6x - 9 = 0$ by Cardon's method.

(b) Solve the equation

$$x^4 + 10x^3 + 26x^2 + 10x + 1 = 0$$

(c) Apply Descartes' rule of signs to find the nature of the roots of the equation

$$x^4 + qx^2 + rx - s = 0 \quad (q,\ r,\ s \text{ being positive})$$

(d) Remove the fractional coefficients of the following :

$$x^3 - \frac{1}{2}x^2 + \frac{2}{3}x - 1 = 0$$ 3+4+3+2=12

★★★

UG Program (under NEP 2020)
1st Semester Exam., 2025 (held in 2026)

MATHEMATICS

(Major)

Paper Code : MT102C

Full Marks : 60

Time : 3 hours

The figures in the margin indicate full marks for the questions

GROUP—A

1. Answer any *six* of the following questions : 2×6=12

(a) If A is an invertible matrix, then prove that A^{-1} is also invertible.

(b) Define orthogonal matrix. Prove that the product of two orthogonal matrices is orthogonal.

(c) State Cayley-Hamilton theorem.

(d) Define linear span. Give an example.

(e) Does the equation

$$x^2 - 5xy + 4y^2 + x + 2y - 2 = 0$$

represent a pair of straight lines?

(f) Find the nature of the conic

$$\frac{8}{r} = 4 - 5\cos\theta$$

(g) Find the polar equation of the left branch of the hyperbola

$$\frac{x^2}{16} - \frac{y^2}{36} = 1$$

(h) Transform to axes inclined at 45° to the original axes the equation $x^2 - y^2 = a^2$ without changing the origin.

GROUP—B

Answer **four** questions, taking **one** from each Unit

UNIT—I

2. (a) Determine the values of α, β, γ when the matrix

$$\begin{bmatrix} 0 & 2\beta & \gamma \\ \alpha & \beta & -\gamma \\ \alpha & -\beta & \gamma \end{bmatrix}$$

is orthogonal.

(b) Define Hermitian matrix. Prove that a necessary and sufficient condition for a matrix A to be Hermitian is that $A = A^*$, where $A^* = [\bar{A}]^T$.

(c) If

$$A + I_3 = \begin{bmatrix} 1 & 3 & 4 \\ -1 & 1 & 3 \\ -2 & -3 & 1 \end{bmatrix}$$

then evaluate $(A + I_3)(A - I_3)$, where I_3 represents 3×3 identity matrix. 4+4+4=12

3. (a) Reduce the following matrix to row-reduced echelon matrix :

$$\begin{pmatrix} 4 & 1 & 2 \\ -3 & 2 & 4 \\ 8 & -1 & -2 \end{pmatrix}$$

(b) Show that the matrix

$$A = \frac{1}{3}\begin{bmatrix} 1 & -2 & 2 \\ -2 & 1 & 2 \\ -2 & -2 & -1 \end{bmatrix}$$

is orthogonal. Using that A is orthogonal, solve the equations, $x - 2y + 2z = 2$, $-2x + y + 2z = -1$ and $2x + 2y + z = 7$.

(c) Express

$$\begin{bmatrix} 1 & 2 & 3 \\ 3 & 4 & 5 \\ 5 & 6 & 7 \end{bmatrix}$$

as the sum of a symmetric and a skew-symmetric matrix. 4+4+4=12

UNIT—II

4. (a) If

$$A = \begin{bmatrix} 1 & 2 \\ -1 & 3 \end{bmatrix}$$

then use Cayley-Hamilton theorem to express $A^6 - 4A^5 + 8A^4 - 12A^3 + 14A^2$ as a linear polynomial in A.

(b) Show that the equations $x + y + z = 6$, $x + 2y + 3z = 14$ and $x + 4y + 7z = 30$ are consistent and solve them.

(c) Let V be a vector space over the field. Prove that the set S of non-zero vectors $\alpha_1, \alpha_2, \alpha_3, \ldots, \alpha_n \in V$ is linearly dependent if and only if some element of S be a linear combination of the others. 4+4+4=12

5. (a) Show that the basis of a vector space need not be unique.

(b) Define sub-space of a vector space. Show that the intersection of two subspaces is always a sub-space of the same. Is the proportion true for union of subspaces? Justify your answer.

(c) Prove that the eigenvectors corresponding to two distinct eigenvalues of a real symmetric matrix are orthogonal. 4+5+3=12

UNIT—III

6. (a) Reduce the equation

$$4x^2 + 4xy + y^2 - 4x - 2y + a = 0$$

to the canonical form and determine the type of the conic represented by it for different values of a.

(b) Find the transformed equation for the equation

$$x^2 - 2xy - 3y^2 + 2x + 14y - 16 = 0$$

when the origin is shifted to the point (1, 2) and the axes are turned through an angle 45°. 6+6

7. (a) If the line $lx + my = n$ be a normal to the ellipse

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

then show that

$$\frac{a^2}{l^2}+\frac{b^2}{m^2}=\frac{(a^2-b^2)^2}{n^2}$$

(b) Find the equation of the diameter of the ellipse $3x^2+4y^2=5$ conjugates to $y+3x=0$.

(c) The polar of the point P with respect to the circle $x^2+y^2=a^2$ touches the circle $4x^2+4y^2=a^2$. Show that the locus of P is the circle $x^2+y^2=4a^2$.

(d) Show that the equation

$$x^2-6xy-27y^2+6x-6y+1=0$$

represents two intersecting straight lines. 3+3+4+2=12

UNIT—IV

8. (a) If

$$ax^2+2hxy+by^2+2gx+2fy+c=0$$

represents a pair of straight lines, then prove that the area of the triangle formed by their bisectors and the axis of x is

$$\frac{\sqrt{(a-b)^2+4h^2}}{2h}\cdot\frac{ca-g^2}{ab-h^2}$$

(b) If the two conics

$$\frac{l_1}{r}=1-e_1\cos\theta \text{ and } \frac{l_2}{r}=1-e_2\cos(\theta-\alpha)$$

touch one another, then show that

$$l_1^2(1-e_2^2)+l_2^2(1-e_1^2)=2l_1l_2(1-e_1e_2\cos\alpha)$$

6+6=12

9. (a) Show that the triangle formed by the lines $ax^2+2hxy+by^2=0$ and $lx+my=1$ is right-angled if

$$(a+b)(al^2+2hlm+bm^2)=0$$

(b) If the conics

$$\frac{l}{r}=1+e\cos\theta \text{ and } \frac{l_1}{r}=1+e\cos(\theta-r)$$

touch at $\theta=\alpha$, then show that

$$l_1=\frac{l(1-e^2)}{e^2+2e\cos\alpha+1}$$

6+6=12

★★★

9. *(a)* Prove that the set of all 2×2 real matrices of the form

$$\begin{bmatrix} x & y \\ -y & x \end{bmatrix}$$

forms a field w.r.t. matrix addition and multiplication.

(b) Show that the set of all integers forms an integral domain but not a field.

(c) Show that the set $\{1, \omega, \omega^2\}$ forms a multiplicative cyclic group, where ω is a cube root of unity. 5+4+3=12

★★★

UG Program (under NEP 2020)
1st Semester Exam., 2025 (held in 2026)

MATHEMATICS

(Minor)

Paper Code : MT101M

Full Marks : 60

Time : 3 hours

The figures in the margin indicate full marks for the questions

GROUP—A

1. Answer any *six* of the following questions : 2×6=12

(a) Define compound statement and contrapositive statement.

(b) If R be a ring such that $a^2 = a$, for all $a \in R$, then prove that

$$a + a = 0,\ \forall\, a \in R$$

(c) Apply Descartes' rule of signs to show that the equation $x^4 + 2x^2 - 7x - 5 = 0$ has two real roots and two non-real roots.

(d) Solve the equation $x^3 - 3x^2 + 4 = 0$, two of its roots being equal.

(e) If a, b and c be any three real numbers, then show that

$$a^2 + b^2 + c^2 \geq ab + bc + ca$$

(f) State the Weierstrass inequalities.

(g) Give an example of a relation which is reflexive and transitive but not symmetric.

(h) "$f(x) = \log x$ is not a mapping from $R \to R$, the set of real numbers." Justify the statement.

GROUP—B

Answer **four** questions, taking **one** from each Unit

UNIT—I

2. (a) Define conjunction and disjunction statements. Write the truth table for the statement $p \wedge (\sim (q \vee r))$.

(b) The relation R in a set A is defined by $2x + 3y = 30$, $x, y \in A$, where

$$A = \{0, 1, 2, 3, 4, \cdots, 15\}$$

Find R. Identify the nature of R with justification.

(c) In the set of real numbers if $f : x \to 2x$, $g : x \to x^2$ and $h : x \to (x+1)$, then find $h \circ (g \circ f)$ and $(h \circ g) \circ f$. (2+4)+3+3=12

3. (a) Prove that $A - (B \cap C) = (A - B) \cup (A - C)$, where A, B and C are subsets of a set X.

(b) If $f : R \to R$ be defined by $f(x) = x^2 + 3$, where R is the set of real numbers, then show that $f^{-1}(7) = \{-2, 2\}$.

(c) Given that $A = R - \{3\}$ and $B = R - \{1\}$, where R is the set of real numbers, and $f : A \to B$ is defined by

$$f(x) = \frac{x-2}{x-3}, \quad x \in A$$

Prove that f is one-one and onto mapping. 5+3+4=12

UNIT—II

4. (a) Solve $x^3 - 3x^2 + 12x + 16 = 0$ by Cardan's method.

(b) If the roots of the equation $x^3 - ax^2 + bx - c = 0$ are in harmonic progression, then show that the mean root is $\frac{3c}{b}$.

(c) If a and b be positive, then show that

$$\left(a + \frac{1}{a}\right)^2 + \left(b + \frac{1}{b}\right)^2 \geq 12\frac{1}{2}$$

where $a + b = 1$. 5+3+4=12

5. (a) Solve the equation $x^4 - 3x^2 - 6x - 2 = 0$ by Ferrari's method.

(b) If α, β and γ be the roots of the equation $x^3 + px^2 + qx + r = 0$ then form the equation whose roots are $\alpha + \frac{1}{\alpha}$, $\beta + \frac{1}{\beta}$ and $\gamma + \frac{1}{\gamma}$.

(c) If a, b and c be positive, then prove that

$$\frac{2}{b+c} + \frac{2}{c+a} + \frac{2}{a+b} > \frac{9}{a+b+c}$$

unless $a = b = c$. 4+4+4=12

UNIT—III

6. (a) Show that the set Z of all integers does not form a group under the operation defined as $x * y = x - y$ for every $x, y \in Z$.

(b) Show that the four roots of unity form an abelian group under ordinary multiplication.

(c) Show that the identity of a subgroup is the same as that of the group. 3+5+4=12

7. (a) Prove that the intersection of any two subgroups of a group $(G, *)$ is again a subgroup of $(G, *)$.

(b) If a be an element of a group G such that $a^2 = a$, then show that $a = e$, the identity.

(c) Show that the set of integers $\{1, 5, 7, 11\}$ forms a group under multiplication modulo 12.

(d) Define centre of a group. 3+3+4+2=12

UNIT—IV

8. (a) If $a = (1\ 2\ 3\ 4)$, then show that the set $\{a, a^2, a^3, a^4\}$ forms a cyclic group.

(b) Prove that every field is an integral domain.

(c) In the ring $(S, +, \cdot)$, S is the set of 2×2 matrices of the form

$$\begin{bmatrix} a & b \\ 0 & c \end{bmatrix}$$

where a, b and c are even integers and $+$ and $\cdot$ are respectively matrix addition and matrix multiplication. Show that $(S, +, \cdot)$ is a non-commutative ring with no unity element. 3+4+5=12

This booklet contains 8 printed pages. **Question Booklet No. :** 26100101

Question Booklet for UG Program (under NEP 2020)
1st Semester Exam., 2025 (held in 2026)

MATHEMATICS

(Inter-Disciplinary)

Full Marks : 60 **Paper Code : MT101ID** **Time : 3 Hours**

Question Booklet **SET No. : A**

SEAL

DO NOT OPEN THIS BOOKLET UNTIL YOU ARE TOLD TO DO SO

Read the following INSTRUCTIONS carefully :

1. **Use black/blue dot pen only.**
2. Fill in the particulars given below in this page.
3. Fill in the particulars (on the **Side 1**) of the OMR Answer Sheet as per Instructions contained in the OMR Answer Sheet.
4. The **SET No.** of this Question Booklet is **A**. Write the SET No. at the specific space provided in the OMR Answer Sheet.
5. There are **60 (sixty)** questions in this Question Booklet, each carrying **1 (one)** mark.
6. Each question or incomplete statement is followed by 4 (four) suggestive answers—[A], [B], [C] and [D] of which only **one** is correct. Mark the correct answer by darkening the appropriate circle.
7. Marking of **more than one** answer against any question will be treated as incorrect response and no mark shall be awarded.
8. **Any change in answer made or erased by using solid or liquid eraser will damage the OMR Answer Sheet resulting in rejection of the whole Answer Sheet by the computer. Therefore, do not change or erase once the answer is marked.**
9. No part of the Question Booklet shall be detached or defaced under any circumstances.
10. ***Use of mobile phone, calculator, log table, compass, scale and any electronic gadget is strictly prohibited in the Examination Hall.***
11. **The OMR Answer Sheet must be returned to the Invigilator before leaving the Examination Hall.**
12. Adoption of unfair means in any form or violation of instruction as mentioned in Point No. **10** shall result in disciplinary action as per rules of the University.
13. The candidate must ensure that the Question Booklet and OMR Answer Sheet are signed by the Invigilator.
14. **After opening the Question Booklet, check the total number of printed pages and report to the Invigilator in case of any discrepancy.**

Roll Number :

OMR Answer Sheet No. :

(As printed in the OMR Answer Sheet)

	Verified and found correct
Full Signature of the Candidate	Signature of the Invigilator with date

SEAL

1. A sentence in Mathematics is classified as a statement if it is

[A] a string of words and symbols having precise meanings

[B] either true or false, but never both or in between

[C] context specific, such as $x+3=7$

[D] always decisively known whether it is true or false

2. A relation which is reflexive, symmetric and transitive is called as

[A] one-one relation

[B] partial order

[C] equivalence relation

[D] systematic relation

3. Which type of memory is volatile and loses its data when power is turned off?

[A] Read-only memory

[B] Flash memory

[C] Random access memory

[D] None of the above

4. Which of the following groups has only output devices?

[A] Scanner, Printer, Monitor

[B] Keyboard, Printer, Monitor

[C] Mouse, Printer, Monitor

[D] Plotter, Printer, Monitor

5. If A is a finite set containing n elements, then the power set $P(A)$ contains

[A] n elements

[B] n^2 elements

[C] 2^n elements

[D] $n+1$ elements

6. Which of the following is ***not*** a set?

[A] Collection of vowels

[B] Collection of odd integers

[C] Collection of good students

[D] Collection of natural numbers

7. Which of the following probabilities ***not*** possible?

[A] $\frac{1}{5}$ [B] $\frac{2}{7}$

[C] $\frac{7}{5}$ [D] $\frac{5}{7}$

8. What is MS PowerPoint?

[A] A video editing software

[B] A spreadsheet software

[C] A word processing software

[D] A presentation software used create slideshows

9. A function $f: M \to N$ is onto, if

[A] f is injective

[B] every element of M has an ima in N

[C] every element of N has a pr image in M

[D] None of the above

10. What is the probability that number selected from the numbe (1, 2, 3, ..., 15) is a multiple of 4?

[A] $\frac{1}{5}$ [B] $\frac{4}{5}$

[C] $\frac{2}{15}$ [D] $\frac{1}{3}$

11. When is the conjunction of two statements, S and T, considered true?

[A] If S is true

[B] If T is true

[C] If at least one of S or T is true

[D] If each of S and T is true

12. If set P has 3 elements and set Q has 4 elements, then the number of relations from P to Q is

[A] 2^3

[B] 2^4

[C] 2^{12}

[D] 3^4

13. If a function $f: R \to S$ is bijective, then

[A] $n(R) > n(S)$

[B] $n(S) > n(R)$

[C] $n(R) = n(S)$

[D] None of the above

14. The shortcut to save a Word document in Windows is

[A] Ctrl + S

[B] Ctrl + C

[C] Ctrl + X

[D] Ctrl + A

15. If $n(A) = 3$ and $n(B) = 5$, then the total number of functions from A to B is

[A] 15

[B] 5^3

[C] ${}^5C_3$

[D] ${}^5P_3$

16. A letter is chosen from the letters in the word 'ENTERTAINMENT'. The probability of choosing a consonant is

[A] $\frac{8}{26}$

[B] $\frac{5}{8}$

[C] $\frac{8}{13}$

[D] None of the above

17. The probability of getting 7 in rolling a fair dice is

[A] 1

[B] 0·5

[C] 0

[D] 0·7

18. Given $P(A) = \frac{1}{4}$, $P(B) = \frac{1}{3}$ and $P(A \cup B) = \frac{1}{2}$. Then $P(A \mid B)$ is equal to

[A] $\frac{1}{12}$

[B] $\frac{1}{4}$

[C] $\frac{1}{3}$

[D] $\frac{3}{4}$

19. If $P(E) = 0{\cdot}30$, then the probability of ***not*** occurring the event E is

[A] 0·30

[B] 0·70

[C] 0·50

[D] 0·60

20. If $A = \{x, y\}$, the power set of A is

[A] $\{\{x\}, \{y\}\}$

[B] $\{\{\phi\}, \{x, y\}\}$

[C] $\{\{\phi\}, \{x\}, \{y\}\}$

[D] None of the above

21. What would be the probability of an event G if H denotes its complement, according to the axioms of probability?

[A] $P(G) = 1 / P(H)$

[B] $P(G) = 1 - P(H)$

[C] $P(G) = 1 + P(H)$

[D] $P(G) = P(H)$

22. What is the full form of HTTPS?

[A] Hypertext Transfer Protocol Secure

[B] Hypertext Transfer Procedure Secure

[C] Hypertext Transfer Process Secure

[D] Hypertext Transfer Policy Secure

23. A class consists of 6 girls and 10 boys. If a committee of 3 is chosen at random from the class, then what is the probability that exactly two girls are selected?

[A] $\frac{3}{14}$

[B] $\frac{27}{56}$

[C] $\frac{15}{56}$

[D] $\frac{1}{28}$

24. A table with all possible values of a random variable and its corresponding probabilities is called

[A] probability mass function

[B] probability density function

[C] cumulative distribution function

[D] probability distribution

25. Which of the following is ***not*** a programming language?

[A] Java

[B] Python

[C] C++

[D] None of the above

26. A function $f : N \to N$ is defined by $f(x) = x^2 + 12$. What is the type of function here?

[A] Bijective

[B] Surjective

[C] Injective

[D] Neither surjective nor injective

27. What is the major use of internet?

[A] Sharing data and information

[B] Browsing Webpages

[C] Sending and receiving emails

[D] All of the above

28. For a function $f : X \to Y$, the set X is called the _____ of f and Y is called the _____ of f.

[A] range, domain

[B] domain, range

[C] domain, codomain

[D] codomain, domain

29. The transmission of a file to our computer from the internet is called

[A] uploading

[B] downloading

[C] receiving file

[D] saving

30. A random variable that can assume any value between two given points is called a/an

[A] discrete random variable

[B] continuous random variable

[C] irregular random variable

[D] uncertain random variable

31. If A and B are independent events, then

[A] A and B^c are independent

[B] A^c and B^c are independent

[C] Both of the above

[D] None of the above

32. What is the formula for independent events?

[A] $P(A \cap B) = P(A)P(B)$

[B] $P(A + B) = P(A)P(B)$

[C] $P(A - B) = P(A)P(B)$

[D] None of the above

33. Which of the following is an open source software operating system?

[A] Microsoft Windows

[B] MAC OS

[C] LINUX

[D] All of the above

34. The negation of the disjunction S or T is

[A] S and T

[B] not S or not T

[C] not S and not T

[D] S only if not T

35. Given that E and F are events such that $P(E) = 0 \cdot 6$, $P(F) = 0 \cdot 3$ and $P(E \cap F) = 0 \cdot 2$, then $P(E \mid F)$ is

[A] $\frac{2}{3}$ [B] $\frac{1}{3}$

[C] $\frac{3}{4}$ [D] $\frac{1}{4}$

36. A die is tossed. If the number is odd, what is the probability that it is prime?

[A] $\frac{2}{5}$ [B] $\frac{2}{3}$

[C] $\frac{3}{5}$ [D] $\frac{5}{6}$

37. How will you define the difference $(B - A)$ of the two sets A and B?

[A] $\{x : x \in A \text{ and } x \notin B\}$

[B] $\{x : x \notin A \text{ and } x \in B\}$

[C] $\{x : x \in A \text{ and } x \in B\}$

[D] $\{x : x \notin A \text{ and } x \notin B\}$

38. A _____ that contains billions of documents in the form of Webpages, is one of the most popular services on the Internet.

[A] Web

[B] Web internet

[C] Telnet

[D] FTP

39. Name of ~ is

[A] negation

[B] conjunction

[C] disjunction

[D] None of the above

40. Which function will you use to enter current time in a worksheet cell?

[A] =current()

[B] =present()

[C] =now()

[D] =today()

41. The probabilities that X, Y and Z will be elected as president of a club are 0·3, 0·5 and 0·2 respectively. The probability that membership fees of club will increase is 0·8 if X is elected president, is 0·1 if Y is elected and is 0·4 if Z is elected. What is the probability that there will be an increase in membership fee? (Use total probability)

[A] 0·36 [B] 0·37

[C] 0·64 [D] 0·15

42. If a random variable can take only integer values between two given points, it is called

[A] continuous random variable

[B] discrete random variable

[C] irregular random variable

[D] uncertain random variable

43. Which of the following relations is the reflexive relation over the set {1, 2, 3, 4}?

[A] {(0,0), (1,1), (2,3), (2,3)}

[B] {(1,1), (1,2), (2,2), (3,3), (4,3), (4,4)}

[C] {(1,1), (1,2), (2,1), (2,3), (3,4)}

[D] {(0,1), (1,1), (2,3), (2,2), (3,4), (3,1)}

44. What is the full form of CPU?

[A] Computer Processing Unit

[B] Computer Principle Unit

[C] Central Processing Unit

[D] Control Processing Unit

45. Who is the father of Computers?

[A] James Gosling

[B] Charles Babbage

[C] Dennis Ritchie

[D] Bjarne Stroustrup

46. If the standard deviation of a data set is zero, then

[A] all observations are zero

[B] all observations are equal

[C] mean is zero

[D] data has large variation

47. If the mean and standard deviation of a data are μ and σ respectively, then mean of $\left(\frac{X-\mu}{\sigma}\right)$ is

[A] μ [B] σ

[C] 1 [D] 0

48. If $f: R \to R$, $g(x) = 3x^2 + 7$ and $f(x) = \sqrt{x}$, then $g \circ f(x)$ is equal to

[A] $3x - 7$ [B] $3x - 9$

[C] $3x + 7$ [D] $3x - 8$

49. Which of the following standard probability density functions is applicable to discrete random variables?

[A] Gaussian distribution

[B] Poisson distribution

[C] Rayleigh distribution

[D] Exponential distribution

50. An injection is a function which is

[A] many-one

[B] one-one

[C] onto

[D] None of the above

51. A software that can be freely accessed and modified is

[A] synchronous software

[B] package software

[C] open-source software

[D] middleware

52. Open-source software can be used for commercial purpose.

[A] True

[B] False

[C] Both of the above

[D] None of the above

53. What is the shortcut key you can press to create a copyright symbol?

[A] Alt+C

[B] Ctrl+C

[C] Alt+Ctrl+C

[D] Ctrl+Shift+C

54. What is the smallest and largest font sizes available in font size tool on formatting toolbar?

[A] 8 and 72

[B] 8 and 64

[C] 12 and 72

[D] 4 and 64

55. Which of the following can only be used in disproving the statements?

[A] Direct proof

[B] Contrapositive proofs

[C] Counter example

[D] Mathematical induction

56. If you need to change the typeface of a document, which menu will you choose?

[A] Edit [B] Format

[C] Tools [D] View

57. What is the default file extension for an Excel workbook?

[A] .xlsx [B] .xls

[C] .txt [D] .csv

58. Which of the following options is ***not*** available in 'Paste Special' dialog box?

[A] Add [B] Subtract

[C] SQRT [D] Divide

59. What is the term used for the value that appears most frequently in a data set?

[A] Mean

[B] Median

[C] Mode

[D] Range

60. Special effects used to introduce slides in a presentation are called as

[A] motion effects

[B] custom animation

[C] transitions

[D] special animation

SPACE FOR ROUGH WORK

SEAL

★★★

UG Program (under NEP 2020)
3rd Semester Exam., 2025 (held in 2026)

MATHEMATICS

(Major)

Paper Code : MT301C

Full Marks : 60

Time : 3 hours

The figures in the margin indicate full marks for the questions

GROUP—A

1. Answer any *six* of the following questions : 2×6=12

4 (a) Form the differential equation for the curve $y^2 = 4a(x+a)$.

(b) Solve $\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 9y = 0$.

(c) Define exact differential equation.

(d) Solve $y = px + p^2 - p$, where $p = \frac{dy}{dx}$.

(e) Find the Wronskian of $y_1 = e^{2t}\cos(5t)$, $y_2 = e^{2t}\sin(5t)$.

(f) Solve $xdx + ydy + \dfrac{xdy - ydx}{x^2 + y^2} = 0$.

(g) Find the integrating factor of the differential equation

$$\frac{dy}{dx} + xy = x^3$$

(h) Find the value of $\dfrac{1}{(D-a)^2}e^{ax}$.

Group—B

Answer *any* **four** questions

2. (a) Solve $\cos^2 x\dfrac{dy}{dx} + y = \tan x$.

(b) Obtain the general solution of the differential equation

$$p^2 - p(e^x + e^{-x}) + 1 = 0$$

where $p = \dfrac{dy}{dx}$.

(c) Find the orthogonal trajectory of $r = ae^{\theta\cot\alpha}$. 4+4+4=12

3. (a) Solve $\dfrac{dy}{dx} + \dfrac{y\log y}{x} = \dfrac{y}{x^2}(\log y)^2$.

(b) Solve $(3x^2y^4 + 2xy)dx + (2x^3y^3 - x^2)dy = 0$.

(c) Solve $y + px = x^4p^2$. 4+4+4=12

4. (a) Solve the following differential equation by the method of undetermined coefficients :

$$\frac{d^2y}{dx^2} + 4y = x^2\sin 2x$$

(b) Solve $\dfrac{d^2y}{dx^2} - 2\dfrac{dy}{dx} + y = xe^{3x}$.

(c) Solve $x^2\dfrac{d^2y}{dx^2} - x\dfrac{dy}{dx} + y = 2\log x$. 4+4+4=12

5. (a) Solve $\dfrac{d^2y}{dx^2} - 2\dfrac{dy}{dx} + 4y = e^x\cos x$.

(b) Solve $x^3\dfrac{d^3y}{dx^3} + 2x^2\dfrac{d^2y}{dx^2} + 2y = 10\left(x + \dfrac{1}{x}\right)$.

(c) Solve $(D^2 - 2D)y = e^x\cos x$ by the method of variation of parameters, where $D \equiv \dfrac{d}{dx}$. 4+4+4=12

6. (a) Reduce the differential equation

$$\frac{d^2y}{dx^2} - 4x\frac{dy}{dx} + (4x^2 - 1)y = -3e^{x^2}\sin 2x$$

into the normal form and hence solve it.

(b) Find the eigenvalues and the corresponding eigen functions of the eigenvalue problem $y'' + \lambda y = 0 \quad (\lambda > 0)$, with $y'(0) = 0$, $y'(\pi) = 0$. 6+6=12

7. (a) Solve the following differential equations by changing the independent variable :

$$\frac{d^2y}{dx^2} - \frac{1}{x}\frac{dy}{dx} + 4x^2 y = x^4$$

(b) Solve and find the singular solution of the differential equation

$$(px - y)(x - py) = 2p$$

where $p = \frac{dy}{dx}$. 6+6=12

8. (a) Solve the following simultaneous differential equations :

$$\frac{dx}{dt} + 4x + 3y = t$$

$$\frac{dy}{dt} + 2x + 5y = e^t$$

(b) Solve

$$y'' - \frac{2}{x}y' + \left(1 + \frac{2}{x^2}\right)y = xe^x$$

by changing the dependent variable.

6+6=12

9. (a) Reduce the equation

$$y'' + 2xy' + (x^2 + 1)y = x^3 + 3x$$

into normal form and hence solve it.

(b) Solve

$$(1 - x^2)\frac{d^2y}{dx^2} + x\frac{dy}{dx} - y = x(1 - x^2)$$

given that $y = x$ is a solution of its reduced equation. 6+6=12

★★★

7. (a) Show that the sequence $\{x_n\}$ defined by $x_1 = \sqrt{2}, x_{n+1} = \sqrt{2x_n}$ for $n \geq 1$, converges to 2. 6

(b) State and prove Leibnitz's theorem of alternating series. 6

8. (a) State Gauss' test for convergence of series of non-negative terms. Test the convergence of the following series : 6

$$\left(\frac{1}{2}\right)^2 + \left(\frac{1\cdot 3}{2\cdot 4}\right)^2 + \left(\frac{1\cdot 3\cdot 5}{2\cdot 4\cdot 6}\right)^2 + \cdots$$

(b) Test the convergence of the series 6

$$1 + \frac{1}{2}x + \frac{2!}{3^2}x^2 + \frac{3!}{4^3}x^3 + \frac{4!}{5^4}x^4 + \cdots$$

9. (a) State and prove root test of infinite series of real numbers. 1+3=4

(b) Define absolute and conditional convergence of a series. Examine the series

$$\sum_{n=2}^{\infty} \frac{(-1)^n}{\log n}$$

for convergence. 1+1+3=5

(c) Test the convergence of the series 3

$$\sum_{n=0}^{\infty} (3 + (-1)^n) z^{2n}$$

★★★

UG Program (under NEP 2020)
3rd Semester Exam., 2025 (held in 2026)

MATHEMATICS

(Major)

Paper Code : MT302C

Full Marks : 60

Time : 3 hours

The figures in the margin indicate full marks for the questions

GROUP—A

1. Answer any *six* of the following : 2×6=12

(a) Prove that

$$\lim_{n\to\infty} n^{\frac{1}{n}} = 1$$

(b) State Bolzano-Weierstrass theorem for set.

(c) If $f(x)$ is an even function and $f'(0)$ exists, show that $f'(0) = 0$.

(d) Examine the continuity at $x = 0$ of the function

$$f(x) = \begin{cases} 2 + x\sin 2x, & -1 \leq x \leq 0 \\ \frac{1}{x}\sin 2x, & 0 < x \leq 1 \end{cases}$$

(e) Find the infimum and the supremum of

$$\left\{\frac{(-1)^n}{n} : n \in N\right\}$$

(f) Define derived set. Obtain the derived set of the set

$$\left\{1+\frac{1}{n} : n \in N\right\}$$

(g) Show that the set

$$S=\left\{1, -1, 1\frac{1}{2}, -1\frac{1}{2}, 1\frac{1}{3}, -1\frac{1}{3} \cdots\right\}$$

is closed.

(h) State limit of a sequence.

GROUP—B

Answer any *four* questions : 12×4=48

2. (a) Define open and closed sets. If G_1 and G_2 are two open sets, then show that $G_1 \cap G_2$ is also open set. 2+4=6

(b) Using the least upper bound (l.u.b.) axioms for real numbers, prove that set of all positive integers is unbounded above. 6

3. (a) Prove that the union of two countable sets is also a countable set. 4

(b) Prove that a set is closed iff its complement is open. 4

(c) Show that **Z** is neither bounded above nor bounded below. 4

4. (a) Prove that arbitrary intersection of closed sets is closed. Is the result true for arbitrary union? Justify your answer. 5+1=6

(b) Show that the set of rational numbers in [0, 1] is countable. 6

5. (a) Show that the sequence $\{x_n\}$, where

$$x_n=\left(1+\frac{1}{n}\right)^n$$

is convergent and that limit of $\left(1+\frac{1}{n}\right)^n$ lies between 2 and 3. 6

(b) Show that the sequence $\{x_n\}$, where

$$x_n=\frac{1}{1!}+\frac{1}{2!}+\frac{1}{3!}+\cdots+\frac{1}{n!}, n \in N$$

is convergent. 6

6. (a) Find all the limit points of the set

$$S=\left\{\frac{(-1)^m}{m}+\frac{1}{n} : m, n=1, 2, 3, 4, \cdots\right\}$$ 6

(b) Show that—

(i) limit of a sequence is unique;

(ii) every bounded sequence is convergent. 3+3=6

(b) Evaluate the integral

$$\int_0^{\log 2}\int_0^{x}\int_0^{x+\log y} e^{x+y+z}\,dz\,dy\,dx$$ 4

(c) Using Green's theorem, evaluate

$$\int_C (x^2 y dx + x^2 dy)$$

where C is the boundary described counter-clockwise of the triangle with vertices $(0, 0)$, $(1, 0)$, $(1, 1)$. 4

9. (a) Evaluate $\int_C \vec{F}\,d\vec{r}$ by Stoke's theorem, where $\vec{F} = y^2\hat{i} + x^2\hat{j} - (x+z)\hat{k}$ and C is the boundary of triangle with vertices at $(0, 0, 0)$, $(1, 0, 0)$ and $(1, 1, 0)$. 4

(b) Evaluate $\iint_S \vec{F}\cdot\hat{n}\,ds$ where

$$\vec{F} = 4xz\hat{i} - y^2\hat{j} + yz\hat{k}$$

where S is the surface of the cube bounded by $x=0$, $x=1$, $y=0$, $y=1$, $z=0$, $z=1$. 4

(c) Evaluate $\iint xy(x+y)\,dx\,dy$ between the regions bounded by $y=x^2$ and $y=x$. 4

★★★

SP

UG Program (under NEP 2020)
3rd Semester Exam., 2024 (held in 2025)

MATHEMATICS

(Minor)

Paper Code : MT301M

Full Marks : 60

Time : 3 hours

The figures in the margin indicate full marks for the questions

GROUP—A

1. Answer any *six* of the following questions : 2×6=12

(a) Convert the triple integral

$$\int_0^2\int_0^{4-x}\int_0^{x+y} dz\,dy\,dx$$

into the order $dz\,dx\,dy$.

(b) What does Stoke's theorem say about a vector field with zero curl?

(c) State Taylor's theorem with Cauchy form of remainder.

(d) If $f(x, y) = e^{xy}$, check whether $f_{xy} = f_{yx}$.

(e) Check whether Rolle's theorem can be applied to $f(x) = |x|$ on $[-1, 1]$

(f) State the conditions for determining maxima and minima of a function of two variables.

(g) Find the Maclaurin's series for $f(x) = xe^x$ up to the x^3 term.

(h) Find the points of inflexion of the function

$$f(x) = x^3 - 6x^2 + 12x - 8$$

GROUP—B

Answer any *four* of the following questions, taking *one* from each Unit : 12×4=48

UNIT—I

2. (a) State and prove Lagrange's mean value theorem. 1+4=5

(b) If $y = \cos(m \sin^{-1} x)$ show that

$$(1-x^2)y_{n+2} - (2n+1)xy_{n+1} + (m^2 - n^2)y_n = 0$$

Also find y_n where $x = 0$. 5

(c) Evaluate : 2

$$\underset{x \to 0}{\text{Lt}} \frac{\tan x - x}{x - \sin x}$$

3. (a) Evaluate : 4

$$\underset{x \to 0}{\text{Lt}} \left(\frac{\sin x}{x}\right)^{1/x}$$

(b) Investigate the continuity of the function 3

$$f(x) = \begin{cases} \dfrac{1-\cos x}{x^2}, & \text{for } x \neq 0 \\ 1, & \text{for } x = 0 \end{cases}$$

(c) Apply mean-value theorem of appropriate order to prove that

$$x > \log(1+x) > x - \frac{1}{2}x^2; \; x > 0$$ 5

UNIT—II

4. (a) Find the equations of those tangents to the circle $x^2 + y^2 = 52$ which are parallel to the line $2x + 3y - 6 = 0$. 4

(b) If ρ, ρ' be the radii of curvature at the ends of two conjugate diameters of an ellipse, prove that

$$(\rho^{2/3} + \rho'^{2/3})(ab)^{2/3} = a^2 + b^2$$ 5

(c) Find all asymptotes of 3

$$xy^2 - yx^2 - x - y - 1 = 0$$

5. (a) Find the asymptotes of

$$x^3 + x^2y - xy^2 - y^3 + x^2 - y^2 = 2$$ 4

(b) Find the envelope of the family of straight lines

$$\frac{x}{a} + \frac{y}{b} = 1$$

where $a^2 + b^2 = c^2$. 4

(c) Expand $\sin x$ in a finite series in powers of x, with remainder in Lagrange's form. 4

UNIT—III

6. (a) Let

$$f(x, y) = \frac{x^2y}{x^4 + y^2}$$

Discuss the existence of the limit of $f(x, y)$ as $(x, y) \to (0, 0)$. 3

(b) Determine whether $(0, 0)$ is a point of extremum for $f(x, y) = y^2 + 2x^2y + 2x^4$. 5

(c) If

$$v = \cos^{-1}\frac{x+y}{\sqrt{x}+\sqrt{y}}$$

then prove that

$$x\frac{\partial v}{\partial x} + y\frac{\partial v}{\partial y} + \frac{1}{2}\cot v = 0$$ 4

7. (a) The roots of the equation in λ

$$(\lambda - x)^3 + (\lambda - y)^3 + (\lambda - z)^3 = 0$$

are u, v, w. Prove that the Jacobian

$$\frac{\partial(u, v, w)}{\partial(x, y, z)} = \frac{-2(y-z)(z-x)(x-y)}{(v-w)(w-u)(u-v)}$$ 5

(b) If $u = e^x \cos y$ and $v = e^x \sin y$, calculate

$$J = \frac{\partial(u, v)}{\partial(x, y)} \text{ and } J' = \frac{\partial(x, y)}{\partial(u, v)}$$

and show that $JJ' = 1$. 4

(c) Show that the function

$$f(x, y) = x^3 + y^3 - 63(x+y) + 12xy$$

is minimum at (3, 3). 3

UNIT—IV

8. (a) Evaluate

$$\iint_E \sqrt{\frac{a^2b^2 - b^2x^2 - a^2y^2}{a^2b^2 + b^2x^2 + a^2y^2}}\, dx\, dy$$

over the positive quadrant of the ellipse

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$ 4

This booklet contains 8 printed pages. **Question Booklet No. :** 26300171

Question Booklet for UG Program (under NEP 2020)
3rd Semester Exam., 2025 (held in 2026)

MATHEMATICS

(Inter-Disciplinary)

Full Marks : 60 **Paper Code : MT301ID** **Time : 3 Hours**

Question Booklet **SET No. : A**

DO NOT OPEN THIS BOOKLET UNTIL YOU ARE TOLD TO DO SO

Read the following INSTRUCTIONS carefully :

1. **Use black/blue dot pen only.**
2. Fill in the particulars given below in this page.
3. Fill in the particulars (on the **Side 1**) of the OMR Answer Sheet as per Instructions contained in the OMR Answer Sheet.
4. The **SET No.** of this Question Booklet is **A**. Write the SET No. at the specific space provided in the OMR Answer Sheet.
5. There are **60 (sixty)** questions in this Question Booklet, each carrying **1 (one)** mark.
6. Each question or incomplete statement is followed by 4 (four) suggestive answers—[A], [B], [C] and [D] of which only **one** is correct. Mark the correct answer by darkening the appropriate circle.
7. Marking of **more than one** answer against any question will be treated as incorrect response and no mark shall be awarded.
8. **Any change in answer made or erased by using solid or liquid eraser will damage the OMR Answer Sheet resulting in rejection of the whole Answer Sheet by the computer. Therefore, do not change or erase once the answer is marked.**
9. No part of the Question Booklet shall be detached or defaced under any circumstances.
10. ***Use of mobile phone, calculator, log table, compass, scale and any electronic gadget is strictly prohibited in the Examination Hall.***
11. **The OMR Answer Sheet must be returned to the Invigilator before leaving the Examination Hall.**
12. Adoption of unfair means in any form or violation of instruction as mentioned in Point No. **10** shall result in disciplinary action as per rules of the University.
13. The candidate must ensure that the Question Booklet and OMR Answer Sheet are signed by the Invigilator.
14. **After opening the Question Booklet, check the total number of printed pages and report to the Invigilator in case of any discrepancy.**

Roll Number :

OMR Answer Sheet No. :

(As printed in the OMR Answer Sheet)

	Verified and found correct
Full Signature of the Candidate	Signature of the Invigilator with date

1. The total number of terms in the expansion of $(x+a)^{51}-(x-a)^{51}$ after simplification is

[A] 102

[B] 25

[C] 26

[D] None of the above

2. If the coefficients of 2nd, 3rd and 4th terms in the expansion of $(1+x)^n$ are in AP, then the value of n is

[A] 2 [B] 7

[C] 11 [D] 14

3. The coefficients of x^n in the expansions of $(1+x)^{2n}$ and $(1+x)^{2n-1}$ are in the ratio

[A] 1 : 2

[B] 1 : 3

[C] 3 : 1

[D] 2 : 1

4. Which term is independent of x in the expansion of $\left(x-\frac{1}{3x^2}\right)^9$?

[A] 3

[B] 4

[C] 5

[D] None of the above

5. Which term of the AP 64, 60, 56, 52, ... is zero?

[A] 15 [B] 16

[C] 17 [D] 18

6. The value of x when $1+6+11+16+\cdots+x=148$ is

[A] 36 [B] 35

[C] 34 [D] 38

7. The arithmetic mean of $a-b$ and $a+b$ is

[A] $a-b$

[B] $a+b$

[C] b

[D] a

8. The 6th term of the GP 2, 6, 18, 54, ... is

[A] 468

[B] 467

[C] 486

[D] 476

9. For what value of x are the numbers $\frac{-2}{7}, x, \frac{-7}{2}$ in GP?

[A] $x=1$ or -1

[B] $x=2$ or -2

[C] $x=3$

[D] $x=-3$

10. If a, b, c are in GP and $a^{1/x}=b^{1/y}=c^{1/z}$, then x, y and z are in

[A] AP

[B] GP

[C] HP

[D] None of the above

11. If ${}^{20}C_{r+1} = {}^{20}C_{r-1}$, then r is equal to

[A] 10 [B] 11

[C] 19 [D] 12

12. Three persons enter into a railway compartment. If there are 5 seats vacant, in how many ways can they take these seats?

[A] 60 [B] 20

[C] 15 [D] 125

13. If ${}^{10}p_r = 5040$, then the value of r is

[A] 3 [B] 5

[C] 2 [D] 4

14. If ${}^{10}C_r = {}^{10}C_{r+4}$, then the value of ${}^{5}C_r$ is

[A] 5 [B] 4

[C] 3 [D] 2

15. If ${}^{k+5}P_{k+1} = \frac{11(k-1)}{2} \cdot {}^{k+3}P_k$, then the values of k are

[A] 7 and 11

[B] 6 and 7

[C] 2 and 11

[D] 2 and 6

16. If ${}^{n}C_r = 120$ and ${}^{n}P_r = 720$, then the value of r is

[A] 3

[B] 4

[C] 5

[D] 6

17. If $a, b > 0$, then according to AM ≥ GM, $\frac{a+b}{2} \geq \sqrt{ab}$, the equality sign holds if

[A] $a = b$

[B] $a > b$

[C] $a < b$

[D] $a = \frac{b}{2}$

18. If $a, b > 0$ and $a + b = 10$, then the maximum value of ab is

[A] 21 [B] 24

[C] 25 [D] 30

19. For any real numbers a_1, a_2, b_1, b_2, the Cauchy-Schwarz inequality states that $(a_1^2 + a_2^2)(b_1^2 + b_2^2) \geq$

[A] $(a_1b_1 + a_2b_2)^2$

[B] $(a_1b_1 - a_2b_2)^2$

[C] $(a_1b_2 + a_2b_1)^2$

[D] None of the above

20. If a, b, c are positive numbers satisfying $a^2 + b^2 + c^2 = 25$, then the maximum value of $a + b + c$ is

[A] 5

[B] $5\sqrt{2}$

[C] $5\sqrt{3}$

[D] 7·5

21. If $E(X) = 3$ and $E(Y) = 5$, then $E(2X + 3Y) = ?$

[A] 21 [B] 19

[C] 18 [D] 20

22. The moment generating function (m.g.f.) of a random variable X is defined as

[A] $M_X(t) = E(t^X)$

[B] $M_X(t) = E(e^{tX})$

[C] $M_X(t) = e^{E(X)t}$

[D] $M_X(t) = tE(X)$

23. $M_X(t) = (0 \cdot 4e^t + 0 \cdot 6)^5$ is the m.g.f. of which distribution?

[A] Binomial

[B] Poisson

[C] Normal

[D] Exponential

24. The first moment about mean is always

[A] positive

[B] negative

[C] zero

[D] undefined

25. The expected value of X^2 is also known as

[A] first moment

[B] second moment about origin

[C] variance

[D] skewness

26. The Poisson distribution is a limiting case of

[A] normal distribution

[B] binomial distribution

[C] exponential distribution

[D] geometric distribution

27. In a Poisson distribution, mean equals

[A] variance

[B] mode

[C] median

[D] standard deviation

28. The m.g.f. of Poisson distribution with mean λ is

[A] $e^{\lambda(e^t - 1)}$

[B] $(1 - \lambda e^t)$

[C] $e^{-\lambda t}$

[D] $e^{t-\lambda}$

29. If X be the binomial variate, the variance is

[A] npq

[B] np

[C] n^2p

[D] npq^2

30. The negative binomial distribution is used when

[A] trials are dependent

[B] number of successes is fixed

[C] number of failures is fixed

[D] probability varies

31. If two variables X, Y are independent, then $E(XY) = ?$

[A] $E(X) + E(Y)$

[B] $E(X)E(Y)$

[C] 0

[D] Undefined

32. Conditional expectation $E(X|Y)$ represents

[A] expectation of X given Y

[B] expectation of Y given X

[C] joint probability

[D] None of the above

33. The sum of probabilities in any discrete distribution is

[A] 0 [B] 1

[C] >1 [D] <1

34. The m.g.f. uniquely determines

[A] mean only

[B] variance only

[C] the entire distribution

[D] None of the above

35. If $X \sim U(a, b)$, then the variance is equal to

[A] $\frac{1}{12}(b-a)^2$

[B] $\frac{1}{12}(b-a)$

[C] $\frac{1}{12}(b+a)$

[D] $\frac{1}{2}(b+a)^2$

36. If X and Y are independent, then $\text{var}(X+Y) =$

[A] $\text{var}(X) + \text{var}(Y)$

[B] $\text{var}(X) - \text{var}(Y)$

[C] $\text{var}(X) \times \text{var}(Y)$

[D] 0

37. The sum of the two independent Poisson variables X_1, X_2 with respective means λ_1, λ_2 follows

[A] Poisson distribution

[B] Normal distribution

[C] Binomial distribution

[D] Uniform distribution

38. The expected value of a discrete uniform distribution over $\{1, 2, ..., n\}$ is

[A] $\frac{n}{2}$

[B] $\frac{(n+1)}{2}$

[C] $\frac{(n-1)}{2}$

[D] $\frac{(n+2)}{2}$

39. The variance of first 50 even natural numbers is

[A] 437

[B] $\frac{437}{4}$

[C] $\frac{833}{4}$

[D] 833

40. Let $x_1, x_2, ..., x_n$ be the values taken by a variable X and $y_1, y_2, ..., y_n$ be the values taken by a variable Y such that $y_i = ax_i + b$, $i = 1, 2, ..., n$. Then

[A] $\text{var}(Y) = a^2 \text{var}(X)$

[B] $\text{var}(X) = a^2 \text{var}(Y)$

[C] $\text{var}(X) = \text{var}(X) + b$

[D] None of the above

41. Which of the following is an example of secondary memory?

[A] RAM

[B] ROM

[C] Hard Disk

[D] Register

42. Which part of the computer performs calculations and comparisons?

[A] Control unit

[B] ALU

[C] Memory unit

[D] Input unit

43. The smallest unit of data in computer is

[A] byte

[B] nibble

[C] bit

[D] word

44. Which of the following is system software?

[A] MS Word

[B] Windows 10

[C] Photoshop

[D] Tally

45. The base of the binary number system is

[A] 2

[B] 8

[C] 10

[D] 16

46. The binary number 11111111 is equivalent to

[A] 127

[B] 255

[C] 256

[D] 511

47. Binary additon of $(1010)_2 + (0101)_2 = ?$

[A] 1110

[B] 1111

[C] 10001

[D] 1100

48. Decimal equivalent of binary 1011 is

[A] 9

[B] 10

[C] 11

[D] 12

49. Which of the following is an example o a loop in C?

[A] for

[B] while

[C] do-while

[D] All of the above

50. What is the size of (char) in a 32-bit compiler?

[A] 1 bit

[B] 2 bits

[C] 1 byte

[D] 2 bytes

51. scanf() is a pre-defined function in which header file?

[A] stdlib.h

[B] ctype.h

[C] stdio.h

[D] stdarg.h

52. What will be the final value of x in the following code?

```
#include<stdio.h>
void main()
{int x=5*9/3+9;
}
```

[A] 3·75

[B] 45

[C] 24

[D] 15

53. C-language was developed in

[A] 1960 [B] 1972

[C] 1980 [D] 1990

54. Which of the following is a valid identifier in C?

[A] 123name

[B] name_1

[C] float

[D] $a-b$

55. The extension of a C source file is

[A] .exe

[B] .c

[C] .cpp

[D] .java

56. Which operator is used for equality comparison?

[A] =

[B] ==

[C] !=

[D] :=

57. In Excel, which symbol is used for formulae?

[A] @

[B] =

[C] #

[D] +

58. The standard header _____ is used for variable list arguments (...) in C.

[A] <stdio.h>

[B] <stdlib.h>

[C] <math.h>

[D] <stdarg.h>

59. In C language, FILE is of which data type?

[A] int

[B] char*

[C] struct

[D] None of the above

60. Which of the following is an operator in C?

[A] /=

[B] sizeof()

[C] ++

[D] All of the above

SPACE FOR ROUGH WORK

SEAL

★★★

UG Program (under NEP 2020)
5th Semester Exam., 2025 (held in 2026)

MATHEMATICS

(Major)

Paper Code : MT501C

Full Marks : 60

Time : 3 hours

The figures in the margin indicate full marks for the questions

GROUP—A

1. Answer any *six* from the following questions : 2×6=12

(a) Find the basic feasible solutions of the system

$$2x_1 + x_2 = 1$$
$$x_2 + x_3 = 2$$

(b) Write down the extreme points, if any, of the set $\{x_1^2 + x_2^2 \le 4,\ x_1 \ge 0,\ x_2 \ge 0\}$.

(c) State complementary slackness theorem.

(d) Define hyperplane.

 (Turn Over)

(e) Find the dual of the following LPP :

$$\text{Minimize } Z = 3x_1 - 2x_2$$

subject to

$$2x_1 + x_2 \le 1$$
$$-x_1 + 3x_2 \ge 4$$
$$x_1, x_2 \ge 0$$

(f) Prove that the solution (2, 3, 2) of the system

$$x_1 + x_2 + 2x_3 = 9$$
$$3x_1 + 2x_2 + 5x_3 = 22$$

is not basic.

(g) State the fundamental theorem of LPP.

(h) Express the vector (7, 2, 3) as a linear combination of the vectors (1, 0, 1) and (2, 1, 0).

GROUP—B

Answer any *four* from the following questions : 12×4=48

2. (a) Find all basic feasible solutions of the following system of equations : 6

$$2x_1 + 3x_2 - x_3 + 4x_4 = 8$$
$$x_1 - 2x_2 + 6x_3 - 7x_4 = -3$$

(b) A manufacturer of medicine is preparing a production plan on medicines *A* and *B*. There are sufficient ingredients available to make 20000 bottles of *A* and 40000 bottles of *B*, but there are only 45000 bottles into which either of the medicines can be put. Furthermore it takes three hours to prepare enough materials to fill 1000 bottles of *A*, it takes one hour to prepare enough materials to fill 1000 bottles of *B* and other are 66 hours available for this operation. The profit is ₹ 8·00 per bottle of *A* and ₹ 7·00 per bottle of *B*.

Formulate this as a linear programming problem to maximize the profit. 4

(c) Prove that a hyperplane is a convex set. 2

3. (a) Solve the following LPP graphically : 4

$$\text{Maximize } Z = 2x_1 - x_2$$

subject to

$$x_1 - x_2 \le 1$$
$$x_1 \le 3$$
$$\text{and } x_1, x_2 \ge 0$$

(b) If x_1 and x_2 are real, show that the set given by $X = \{(x_1, x_2) \mid 9x_1^2 + 4x_2^2 \le 36\}$ is a convex set. 4

(c) Whether the set of all convex combinations of a finite number of points is a convex set or not? Justify. 1+3=4

4. (a) Prove that a basic feasible solution of an LPP corresponds to an extreme point of the convex set of all feasible solutions. 6

(b) (2, 1, 3) is a feasible solution of the set of equations—

$$4x_1 + 2x_2 - 3x_3 = 1, \quad -6x_1 - 4x_2 + 5x_3 = -1$$

Reduce it to a basic feasible solution of the set. 6

5. (a) Use Charne's Big-M method to solve the following LPP : 6

$$\text{Maximize } Z = x_1 + 5x_2$$

subject to the constraints

$$3x_1 + 4x_2 \leq 6$$
$$x_1 + 3x_2 \geq 3$$
$$\text{with } x_1, x_2 \geq 0$$

(b) Prove that the dual of the dual of a primal LPP is the primal itself. 6

6. (a) Formulate the dual of the following LPP : 6

$$\text{Maximize } Z = 2x_1 + 3x_2 + 4x_3$$

subject to

$$x_1 - 5x_2 + 3x_3 = 7$$
$$2x_1 - 5x_2 \leq 3$$
$$3x_1 - x_3 \geq 5$$

$x_1, x_2 \geq 0$ and x_3 is unrestricted in sign.

(b) Show that the LPP

$$\text{Maximize } Z = 4x_1 + 14x_2$$

subject to

$$2x_1 + 7x_2 \leq 21$$
$$7x_1 + 2x_2 \leq 21$$
$$x_1, x_2 \geq 0$$

has alternate optimal solutions. Find two solutions of the LPP. 6

7. (a) Solve the following LPP : 6

$$\text{Maximize } Z = 5x_1 - 2x_2 + 3x_3$$

subject to

$$2x_1 + 2x_2 - x_3 \geq 2$$
$$3x_1 - 4x_2 \leq 3$$
$$x_2 - 3x_3 \leq 5$$
$$x_1, x_2, x_3 \geq 0$$

(b) Obtain an optimal basic feasible solution to the following transportation problem : 6

	D_1	D_2	D_3	D_4	
O_1	19	30	50	10	7
O_2	70	30	40	60	9
O_3	40	08	70	20	18
	5	8	7	14	

8. (a) By solving the dual of the following problem show that the given problem has no feasible solution : 6

$$\text{Minimize } Z = x_1 - x_2$$

subject to

$$2x_1 + x_2 \geq 2$$
$$-x_1 - x_2 \geq 1$$
$$x_1, x_2 \geq 0$$

(b) Find the minimum cost of the assignment problem whose cost coefficients are as given below : 6

	I	II	III	IV
1	4	5	3	2
2	1	4	−2	3
3	4	2	1	5

(Continued)

9. (a) Solve the following LPP by two-phase method : 6

$$\text{Maximize } Z = 5x_1 + 3x_2$$

subject to

$$2x_1 + x_2 \leq 1$$
$$3x_1 + 4x_2 \geq 12$$
$$x_1, x_2 \geq 0$$

(b) Use dual simplex method to solve the following LPP : 6

$$\text{Minimize } Z = 6x_1 + 7x_2 + 3x_3 + 5x_4$$

subject to

$$5x_1 + 6x_2 - 3x_3 + 4x_4 \geq 12$$
$$x_2 + 5x_3 - 6x_4 \geq 10$$
$$2x_1 + 5x_2 + x_3 + x_4 \geq 8$$
$$x_1, x_2, x_3, x_4 \geq 0$$

★★★

UG Program (under NEP 2020)
5th Semester Exam., 2025 (held in 2026)

MATHEMATICS

(Major)

Paper Code : MT502C

Full Marks : 60

Time : 3 hours

The figures in the margin indicate full marks for the questions

GROUP—A

Answer *any* **six** questions : 2×6=12

1. (a) Determine if the permutation

$$\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 5 & 3 & 4 & 6 & 1 & 2 \end{pmatrix}$$

is odd or even.

(b) Let H be a sub-group of G. Show that any coset of H other than H is not a sub-group of G.

(c) Define kernel of a group homomorphism.

(d) If S is an ideal of a ring R with unity and $1 \in S$, prove that $S=R$.

(e) State the first isomorphism theorem on groups.

(f) If $a \equiv b \pmod{m}$ and $x \equiv y \pmod{m}$, then prove that $ax \equiv by \pmod{m}$.

(g) Find $\phi(15)$ and $\phi(19)$ where ϕ denotes Euler's phi function.

(h) State Goldbach conjecture.

GROUP—B

Answer *any* **four** questions : 12×4=48

2. (a) Prove that any non-identity permutation $\alpha \in S_n$ $(n \geq 2)$ can be expressed as a product of disjoint cycles, where each cycle is of length ≥ 2.

(b) If H is a sub-group of G such that $[G:H]=2$, prove that H is a normal sub-group.

(c) Show that the intersection of any two normal sub-groups of a group G is a normal sub-group. 4+4+4=12

3. (a) Prove that every finite group G is isomorphic to a permutation group.

(b) Prove that any two left cosets of a sub-group are either disjoint or identical.

(c) State and prove Lagrange's theorem for groups. 4+4+4=12

4. (a) State and prove Cauchy's theorem for finite Abelian group.

(b) Show that every sub-group of an Abelian group is normal.

(c) Define ideal of a ring. Show that the intersection of two ideals of a ring is an ideal of that ring. 4+4+(1+3)=12

5. (a) Prove that any two right cosets of a sub-group are either disjoint or identical.

(b) State and prove the fundamental theorem on homomorphism of groups.

(c) Prove that every homomorphic image of a commutative ring is a commutative ring. 4+5+3=12

6. (a) Define kernel of a ring homomorphism. If ϕ is a homomorphism of a ring R into a ring R' with kernel S, then show that S is an ideal of R.

(b) If $f: G \to G'$ is an isomorphism of groups, then show that the order of an element $a \in G$ is equal to the order of the f-image of a.

(c) Find the sum and product of the following polynomials over the ring $(I_6, +_6, \cdot_6)$:

$$f(x) = 5x^2 + 3x + 2, \; g(x) = 2x^3 + 5x + 3$$

Also find degree. 5+4+3=12

7. (a) State and prove Fermat's little theorem.

(b) If $ac \equiv bc \pmod{m}$ and $d = (c, m)$, then prove that $a \equiv b\left(\bmod \frac{m}{d}\right)$.

(c) Find the integers m and n such that $d = mx + ny$, where $x = 540$, $y = 168$ and $d = \gcd(x, y)$. (1+3)+4+4=12

8. (a) Use Chinese remainder theorem to solve the simultaneous system of linear congruences :

$$x \equiv 2 \pmod 3$$
$$x \equiv 3 \pmod 5$$
$$x \equiv 2 \pmod 7$$

(b) Find the g.c.d. of 256 and 1166 and hence express the g.c.d. as a linear combination of 256 and 1166.

(c) Prove that the equation $ax + by = c$ has an integral solution iff (a, b) divides c ($a \neq 0$, $b \neq 0$ and c are integers). 4+4+4=12

9. (a) Define quotient ring. If ϕ is a homomorphism of a ring R into a ring R', then show that $\phi(R)$ is a subring of R'.

(b) If f is a homomorphism of a group G into a group G' with kernel k, then show that k is a normal subgroup of G.

(c) If f be mapping from $(Z, +)$ to $(G, \cdot)$ where $G \equiv \{1, -1\}$ and f is defined by

$$f(x) = \begin{cases} 1, & \text{when } x \text{ is even} \\ -1, & \text{when } x \text{ is odd} \end{cases}$$

show that f is a homomorphism. Is f an isomorphism? 4+4+4=12

★★★

(b) Show that $-1 \le r \le 1$, where r is the correlation coefficient of two random variables X and Y.

(c) If X and Y are two independent random variables, then show that

$$E(XY) = E(X)E(Y)$$

4+4+4=12

★★★

UG Program (under NEP 2020)
5th Semester Exam., 2025 (held in 2026)

MATHEMATICS

(Major)

Paper Code : MT503C

Full Marks : 60

Time : 3 hours

The figures in the margin indicate full marks for the questions

Ordinary calculator is allowed

Group—A

1. Answer any *six* of the following questions : 2×6=12

(a) Let the mean of four numbers 3, 7, $2x$ and $y (x > y)$ be 5. Then find the mean of four numbers $3+2x$, $7+2y$, $x+y$ and $x-y$.

(b) Give an example to show that two dependent random variables can be uncorrelated.

(c) Show that the correlation coefficient between two random variables is the geometric mean between the two linear regression coefficients.

(d) What will be the angle between the lines of regression, if (i) the variables are uncorrelated and (ii) the variables are perfectly correlated?

(e) Find the moment-generating function of uniform distribution.

(f) Give axiomatic definition of probability.

(g) When are two events said to be independent? Show that if the events A, B are independent, then so are their complements.

(h) Calculate the mean deviation about mode from the following :

Value (x)	5	6	7	8	9	Total
Frequency (f)	1	4	7	6	2	20

GROUP—B

Answer any *four* of the following questions : 12×4=48

2. (a) Find the quartile deviation from the following frequency table :

Marks	10–19	20–29	30–39	40–49	50–59	60–69	Total
Frequency	8	11	15	17	17	7	75

(b) The first four moments of a distribution about the point 5 are 2, 20, 40 and 50 respectively. Find the mean, variance, third and fourth central moments and the measures of skewness and kurtosis of the distribution. 6+6=12

3. (a) Find the second, third and fourth central moments of the marks x_i having frequency f_i ($i = 1, 2, 3, 4, 5, 6, 7$), given that

$$u_i = \frac{x_i - 127\cdot45}{5}, \ \sum f_i = 100, \ \sum u_i = 0,$$

$$\sum f_i u_i = -20, \ \sum f_i u_i^2 = 220, \ \sum f_i u_i^3 = -50,$$

$$\sum f_i u_i^4 = 1240$$

Hence, find the measures of skewness and kurtosis.

(b) The mean and standard deviation of 20 items are found to be 10 and 2 respectively. At the time of checking, it was found that one item 8 was incorrect. Calculate the mean and standard deviation, if (i) the wrong item is omitted and (ii) it is replaced by 12. 6+(3+3)=12

4. (a) Define binomial distribution. Find the moment-generating function of binomial distribution with parameters n and p. Hence deduce mean and variance of binomial distribution.

(b) Given that variance of x is 9, the equation of line of regression of y on x is $8x - 10y + 66 = 0$ and the equation of line of regression of x on y is $40x - 18y = 214$. Find (i) the mean of x and the mean of y, (ii) correlation coefficient between x and y, (iii) standard deviation of y. (1+3+2)+(2+2+2)=12

5. *(a)* Fit a second-degree polynomial of the form $y = a + bx + cx^2$ to the following data by the method of least squares :

x	1	1·5	2	2·5	3	3·5	4
y	1·1	1·3	1·6	2	2·7	3·4	4·1

(b) State and prove Bayes' theorem.

6+(2+4)=12

6. *(a)* Derive the equations of the lines of regression by the method of least squares.

(b) Show that the regression coefficients are independent of the change of origin but not of scale.

(c) Define Pearson correlation coefficient.

6+4+2=12

7. *(a)* Find the mean and variance of normal distribution with parameters μ and σ^2.

(b) Find the characteristic function of a random variable which follows *(i)* binomial (n, p) distribution, *(ii)* Poisson distribution with parameter μ, *(iii)* normal distribution with mean μ and variance σ^2.

(3+3)+(2+2+2)=12

8. *(a)* Let

$$f(x) = \begin{cases} \frac{1}{2}, & -1 < x < 1 \\ 0, & \text{elsewhere} \end{cases}$$

be the p.d.f. of the random variable X. Find the distribution function and probability density function of $Y = X^2$.

(b) If A and B are two events such that

$$P(A) = 0 \cdot 7, \; P(B) = 0 \cdot 4, \; P(A \cap \bar{B}) = 0 \cdot 5$$

where $\bar{B}$ denotes the complement of B, then find the value of $P(B \mid (A \cup \bar{B}))$.

(c) Bag B_1 contains 6 white and 4 blue balls, bag B_2 contains 4 white and 6 blue balls and bag B_3 contains 5 white and 5 blue balls. One of the bags is selected at random and a ball is drawn from it. If the ball is white, then find the probability that the ball is drawn from the bag B_2.

4+4+4=12

9. *(a)* In a binomial distribution $B(n, p)$, the sum and the product of the mean and the variance are 5 and 6 respectively, then find the value of $6(n + p - q)$.

8. (a) Define fundamental tensor. Show that the fundamental tensor g_{ij} is a symmetric tensor of type (0, 2).

(b) If $a_{ij}(\neq 0)$ are the components of a covariant tensor of order 2 such that $b a_{ij} + c a_{ji} = 0$, where b and c are non-zero scalars, then show that either $b = c$ and a_{ij} is a skew-symmetric or $b = -c$ and a_{ij} is symmetric.

(c) If a_{ij} is a skew-symmetric tensor, then prove that $(\delta^i_j \delta^k_l + \delta^i_l \delta^k_j) a_{ik} = 0$.

(1+4)+5+2=12

9. (a) Define covariant and contravariant vectors.

(b) If $A^k_{ij} B^{il}_k = 0$ for every B^{jl}_k, then show that A^k_{ij} vanishes identically.

(c) Prove that

$$\frac{\partial g^{mk}}{\partial x^l} = -g^{mi}\begin{Bmatrix} k \\ il \end{Bmatrix} - g^{ki}\begin{Bmatrix} m \\ il \end{Bmatrix}$$

2+5+5=12

★★★

UG Program (under NEP 2020)
5th Semester Exam., 2025 (held in 2026)

MATHEMATICS

(Major)

Paper Code : MT504C

Full Marks : 60

Time : 3 hours

The figures in the margin indicate full marks for the questions

Group—A

1. Answer any *six* from the following questions : 2×6=12

(a) Two particles are connected by a light inextensible string passing over a smooth fixed pulley. If the system moves with an acceleration of $16\ \text{ft/sec}^2$, compare the masses of the particles.

(b) The velocity of a particle moving in a straight line at any time t, when its distance from the origin is x, is given by $x = \frac{1}{2}v^2$. Show that the acceleration of the particle is constant.

(c) Define an apse and apsidal distance.

(d) State Kepler's laws of planetary motion.

(e) The position of a moving particle at time t is given by $x = a\cos nt$, $y = a\sin nt$. Find its velocity and acceleration.

(f) In an SHM, f be the acceleration and v the velocity at any instant and T is the periodic time, then prove that $f^2T^2 + 4\pi^2v^2$ is constant.

(g) If A^{ij} is a skew-symmetric tensor, then show that $A^{jk}\{^{i}_{jk}\} = 0$.

(h) Prove that $[ij, k] + [kj, i] = \frac{\partial g_{ik}}{\partial xj}$; symbols have their usual meanings.

GROUP—B

Answer *any* **four** questions

2. (a) A point moves in a straight line so that its distance s from a fixed point at time t is proportional to t^n. If v be the velocity and f the acceleration at any time t, then show that $(n-1)v^2 = nfs$.

(b) Find the expression for radial and transverse components of acceleration of a particle in plane curvilinear motion in terms of its polar coordinates.

(c) A particle moves freely in a parabolic path given by $y^2 = 4ax$, under a force always perpendicular to its axis. Find the law of force. 4+5+3=12

3. (a) A particle is performing a simple harmonic motion of period T about a centre O and it passes through a point P where $OP = b$ with velocity v in the direction OP. Prove that the time which elapses before it returns to P is

$$\frac{T}{\pi}\tan^{-1}\left(\frac{vT}{2\pi b}\right)$$

(b) A particle describes an elliptic orbit under a force which is always directed towards the centre of the ellipse. Find the law of force.

(c) A particle describes an equiangular spiral $r = ae^{\theta}$ in such a manner that its acceleration has no radial component. Prove that its angular velocity is constant and the magnitude of the velocity and the acceleration are each proportional to r. 5+4+3=12

4. (a) In a central orbit, with usual notations, prove that

$$v \propto \frac{1}{P} \text{ and } v^2 = \lambda^2\left[u^2 + \left(\frac{du}{d\theta}\right)^2\right]$$

(b) Find the law of force to the pole when the path is the cardioid $r = a(1 - \cos\theta)$ and prove that if F be the force at the apse, and v the velocity $3v^2 = 4aF$.

(c) If v_1 and v_2 are the linear velocities of a planet when it is respectively nearest and farthest from the sun, prove that $(1-e)v_1 = (1+e)v_2$. Where e is the eccentricity of the planet's orbit. 4+4+4=12

5. (a) A particle moves with a central acceleration which varies inversely as the cube of the distance. If it be projected from an apse at a distance a from the origin with a velocity which is equal to $\sqrt{2}$ times the velocity for a circle of radius a, then show that the equation of the path is $r\cos\left(\frac{\theta}{\sqrt{2}}\right) = a$.

(b) A particle of mass m is projected vertically upwards under gravity, the resistance of the air being mk times the velocity. Show that the greatest height attained by the particle is

$$\frac{V^2}{g}[\lambda - \log(1+\lambda)]$$

where V is the terminal velocity of the particle and λV is the initial velocity.

(c) A particle describes an equiangular spiral, whose pedal equation is $p = r\sin\alpha$, under a force to the pole. Show that the force varies inversely with r^3. 5+5+2=12

6. (a) A particle moves with a central acceleration $\mu\left(r + \frac{a^4}{r^3}\right)$, being projected from an apse at a distance a with a velocity $2\sqrt{\mu a}$, prove that its path is $r^2(2 + \cos\sqrt{3}\theta) = 3a^2$.

(b) A body projected with an initial velocity u_0 at a height h above the surface of the earth becomes a satellite with circular orbit. Show that

$$u_0 = a\sqrt{\frac{g_0}{a+h}}$$

where g_0 is the acceleration due to gravity on the surface of the earth and a is its radius. 6+6=12

7. (a) State and prove the quotient law for tensors of type (0, 1).

(b) If $B_{ij} = A_{ji}$, where A_{ij} is a covariant tensor, then show that B_{ij} is a tensor of order 2.

(c) If $A = A_i\alpha^i$, then show that $\frac{\partial A}{\partial x^j} = A_{ij}\alpha^i$. 4+4+4=12

UG Program (under NEP 2020)
5th Semester Exam., 2025 (held in 2026)

MATHEMATICS
(Minor)

Paper Code : MT501M

Full Marks : 60

Time : 3 hours

The figures in the margin indicate full marks for the questions

Scientific Calculator is allowed

GROUP—A

1. Answer any *six* questions from the following : $2\times6=12$

(a) State Archimedean property of real numbers (any one version).

(b) Define uncountable sets and give an example.

(c) Define bounded below sequence with example.

(d) Show that the sequence $\left\{1+\frac{1}{n}\right\}$ converges to the limit 1.

(e) What is monotonic sequence? Give example.

(f). Compute $f(1{\cdot}25)$ from the following table :

x	1·1	1·2	1·3
$f(x)$	7·8	8·7	9·6

(g) If $y = 3x^7 - 6x$, find the percentage error in y at $x = 1$ if the error in x is 0·05.

(h) What is shift operator?

GROUP—B

Answer any *four* questions from the following : 12×4=48

2. (a) Prove that the arbitrary union of open sets is open. 3

(b) State and prove Bolzano-Weierstrass theorem for sets. 1+4=5

(c) Let S be a non-empty subset of $\mathbb{R}$ and S is bounded above. Prove that an upper bound u of S is the supremum of S iff for each $\varepsilon > 0 \ \exists \ s \in S$ such that $u - \varepsilon < s \le u$. 4

3. (a) Prove that finite union of closed sets is closed. 4

(b) Show that the set of rational numbers in $[0,1]$ is countable. 4

(c) Prove that the intersection of any finite number of open sets is open. 4

4. (a) Test the convergence of the series

$$2x + \frac{3x^2}{8} + \frac{4x^3}{27} + \ldots + \frac{(n+1)x^n}{n^3} + \ldots$$ 4

(b) Prove that a monotonic decreasing sequence, which is bounded below, converges to its exact lower bound. 4

(c) Show that

$$\lim_{n\to\infty}\left[\frac{1}{\sqrt{n^2+1}} + \frac{1}{\sqrt{n^2+2}} + \ldots + \frac{1}{\sqrt{n^2+n}}\right] = 1$$ 4

5. (a) State and prove Cauchy's convergence criterion for sequence. 1+4=5

(b) Use comparison test to examine the convergence of

$$\sum_{n=1}^{\infty}\left(\frac{1}{\sqrt{n}} + \frac{1}{\sqrt{n-1}}\right)$$ 3

(c) Test the convergence of the following series : 4

$$\sum \frac{n^2-1}{n^2+1} x^n, \ x > 0$$

6. (a) Compute $f(0{\cdot}33)$ using the following table : 5

x	0·3	0·32	0·34	0·36	0·38	0·4
$f(x)$	1·7196	1·7698	1·7804	1·7912	1·8024	1·8139

(b) Find, by Lagrange interpolation formula, the interpolation polynomial which corresponds to the following data : 5

x	−1	0	2	5
$f(x)$	9	5	3	15

(c) Find the relative error in the computation of $x-y$ for $x = 12{\cdot}05$ and $y = 8{\cdot}02$ having absolute errors $\Delta x = 0{\cdot}005$ and $\Delta y = 0{\cdot}001$. 2

7. (a) Prove that the nth order forward difference of a polynomial of degree n is constant and higher orders vanish. 4

(b) Evaluate $\int_0^1 x^3 dx$ by trapezoidal rule with $n = 5$. 4

(c) Calculate the value of $\int_0^1 \frac{x}{1+x} dx$ correct up to three significant figures, taking six intervals by Simpson's one-third rule. 4

8. (a) If $y = 4x^6 - 5x$, find the percentage error in y at $x = 1$ if the error in x is 0·04. 2

(b) Compute $f'(1{\cdot}1)$ and $f''(1{\cdot}1)$ from the following table : 5

x	1·1	1·2	1·3	1·4	1·5
$f(x)$	2·0091	2·0333	2·0692	2·1143	2·1667

(c) Evaluate $\int_0^{0{\cdot}6} \frac{dx}{\sqrt{1-x^2}}$ using Simpson's $\frac{3}{8}$th rule. 5

9. (a) From the following table, find the value of y when $x = 1{\cdot}54$: 5

x	1·0	1·1	1·2	1·3	1·4	1·5
$y = f(x)$	0·24197	0·21785	0·19419	0·17137	0·14973	0·12952

(b) Find the first-order derivatives of the function $f(x)$ tabulated below at the points $x = 1{\cdot}5$ and $x = 4{\cdot}0$: 2½+2½=5

x	1·5	2·0	2·5	3·0	3·5	4·0
$f(x)$	3·375	7·000	13·625	24·000	38·875	59·000

(c) If $\frac{5}{6}$ be represented approximately by 0·8333, find *(i)* relative error and *(ii)* percentage error. 1+1=2

★★★